AD-A238 217

UMENTATION PAGE

2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
June 1991	Technical Report

4. TITLE AND SUBTITLE	5. FUNDING NUMBERS
Closed Loop Vibrational Control: State and Output Feedback Stabilizability	DAAL03-90-G-0004

6. AUTHOR(S)

S.M. Meerkov and E.-K. Poh

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
University of Michigan Ann Arbor, MI 48109-2140	

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY REPORT NUMBER
U. S. Army Research Office P. O. Box 12211 Research Triangle Park, NC 27709-2211	ARO 27510.2-EG

11. SUPPLEMENTARY NOTES

The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
Approved for public release; distribution unlimited.	

13. ABSTRACT (Maximum 200 words)

Linear plants with control signals entering the open loop dynamics as amplitudes of periodic, zero average functions are considered. Stabilizability properties of such plants by state space and output feedback controllers are analyzed.

14. SUBJECT TERMS			15. NUMBER OF PAGES
			13
			16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

CLOSED LOOP VIBRATIONAL CONTROL: STATE AND OUTPUT FEEDBACK STABILIZABILITY

P. Γ. KABAMBA

Department of Aerospace Engineering

The University of Michigan

Ann Arbor, MI 48109-2140

S.M. MEERKOV and E.-K. POH

Department of Electrical Engineering and Computer Science

The University of Michigan

Ann Arbor, MI 48109-2122

Abstract

Linear plants with control signals entering the open loop dynamics as amplitudes of periodic, zero average functions are considered. Stabilizability properties of such plants by state space and output feedback controllers are analyzed.

91   1   093

1. INTRODUCTION AND PROBLEM STATEMENT

Consider the following problem: Given the system

$$\dot{x} = Ax + Buf(t) \ ,$$

$$x \in \mathbb{R}^n, \ u \in \mathbb{R}^m, \ y \in \mathbb{R}^P \ , \tag{1.1}$$

$$f(t) \in \mathbb{R}, \ f(t) = f(t+T), \ T \neq 0 \ ,$$

$$\frac{1}{T} \int_0^T f(t)dt = 0 \ ,$$

find a state space controller $K \in \mathbb{R}^{m \times n}$ such that with

$$u = Kx \ , \tag{1.2}$$

the closed loop system

$$\dot{x} = (A + BKf(t))x \tag{1.3}$$

has desired dynamic properties.

A characteristic feature of this problem is that the control, u, enters the open loop dynamics as an amplitude of a periodic, zero average function, and this amplitude can be chosen as a function of the state or, more generally, output. Systems of this form arise in a number of practical applications. One of these is the Higher Harmonic Control (HHC) in helicopters. Here a periodic feathering of rotor blades around a fixed pitch angle is introduced in order to suppress the fuselage vibrations. The amplitude of the vibrations is typically chosen as a function of the acceleration of pilot's seat (system output). Recent experiments [1]-[3] have shown that HHC may lead to an order of magnitude reduction in fuselage vibrations. However, no formalized methods for HHC controllers design have been described in the literature.

Another example is the periodic operation of chemical reactors [4]. Here again the problem is to choose the amplitude of input flow vibrations so that the closed loop system behaves as desired.

From the theoretical standpoint, problem (1.1), (1.2) is closely related to the problem of vibrational control [5]-[7] which can be formulated as follows: Given $A \in \mathbb{R}^{n \times n}$ find a periodic, zero average matrix $B(t) \in \mathbb{R}^{n \times n}$ such that

$$\dot{x} = (A + B(t))x \qquad (1.4)$$

is asymptotically stable. The only difference is that (1.4) does not contain structural constraints imposed by the feedback whereas (1.3) does. Nevertheless, due to the obvious similarities between (1.3) and (1.4) (including the methods of their analysis–see below), the problem (1.1), (1.2) will be referred to as **closed loop vibrational control**. The effect of the structural contraints on the problem of vibrational stabilizability is the topic of this note.

The results presented below differ also from those of [8] in that the latter uses the vibrations introduced in the parameters of dynamic output controllers whereas the plants are time invariant.

All three cases however, i.e. [5]-[7], [8] and the present work, involve linear systems with periodic coefficients. In order to simplify the analysis and obtain constructive results, following [5]-[8], we assume that the periodic function $f(t)$ is of high frequency as compared with the dynamics of $\dot{x} = Ax$. Formally, this means that function f has the asymptotic form $f(\frac{t}{\epsilon})$ where $0 < \epsilon << 1$, is sufficiently small. Thus, more precisely the problem addressed in this note is as follows:

Given

$$
\begin{aligned}
\dot{x} &= Ax + Buf\left(\frac{t}{\epsilon}\right), \\
y &= Cx,
\end{aligned}
\qquad (1.5)
$$

$$x \in \mathbb{R}^n, \ u \in \mathbb{R}, \ y \in \mathbb{R}, \ f \in \mathbb{R},$$

$f(\cdot)$ periodic, zero average, $0 < \epsilon << 1$ small parameter, determine under what conditions

there exists a time invariant state space controller

$$u = Kx \ , \tag{1.6}$$

or a time invariant output controller

$$
\begin{aligned}
u &= K\hat{x} \ , \\
\dot{\hat{x}} &= A\hat{x} + Buf\left(\frac{t}{\epsilon}\right) + L(y - \hat{y}) \ , \\
\hat{y} &= C\hat{x} \ ,
\end{aligned}
\tag{1.7}
$$

such that the closed loop dynamics are asymptotically stable. The feedback laws (1.6), (1.7) are restricted to be time invariant for reasons of practical implementation and possible uncertainty in the knowledge of $f(\cdot)$. Problem (1.5), (1.6) is considered in Section 2 and problem (1.5), (1.7) is discussed in Section 3. In addition, we characterize the pole placement capabilities ensured by closed loop vibrational control and present the corresponding results in Section 4. The conclusions are formulated in Section 5.

2. STATE SPACE FEEDBACK

Theorem 2.1: There exists $\epsilon_0 > 0$ such that for all $0 < \epsilon \leq \epsilon_0$ system (1.5) is stabilizable by a state space feedback (1.6) if and only if (A, B) is stabilizable and the sum of all the controllable eigenvalues of A is negative.

Proof: Necessity is proved by the following considerations. The state model in (1.5) has the Kalman controllable form

$$
\begin{bmatrix} \dot{x}_c \\ \dot{x}_{nc} \end{bmatrix} = \begin{bmatrix} A_c & A_{12} \\ 0 & A_{nc} \end{bmatrix} \begin{bmatrix} x_c \\ x_{nc} \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} uf\left(\frac{t}{\epsilon}\right) \ . \tag{2.1}
$$

Since A_{nc} is not affected by feedback, the stabilizability of (A, B) is necessary.

Consider the controllable part of (2.1):

$$\dot{x}_c = A_c x_c + B_1 uf\left(\frac{t}{\epsilon}\right) \ , \quad x_c \in \mathbb{R}^m, \ u \in \mathbb{R} \ . \tag{2.2}$$

3

Introducing a state feedback $u = Kx_c$, we obtain

$$\dot{x}_c = \left(A_c + B_1 K f\left(\frac{t}{\epsilon}\right)\right) x_c \ . \tag{2.3}$$

Since (2.3) is periodic, there exists a Lyapunov transformation which reduces (2.3) to an equation with constant coefficients,

$$\dot{z} = \Lambda z \ ,$$

preserving the stability property. The following equality is true [5]:

$$\frac{1}{T}\int_0^T \mathrm{Tr}\left[A_c + B_1 K f\left(\frac{t}{\epsilon}\right)\right] dt = \mathrm{Tr}\ \Lambda$$

where T is the period of $f(t/\epsilon)$. Thus

$$\mathrm{Tr}\ A_c = \mathrm{Tr}\ \Lambda \ ,$$

where $\mathrm{Tr}\ A_c$ is equal to the sum of all the controllable eigenvalues. This completes the proof of necessity.

The suffiency is proved as follows: Consider the Kalman controllable form (2.1) of the system (1.5) where all the eigenvalues of A_{nc} have negative real parts. Without loss of generality, assume that (2.2) is in the controllable canonical form i.e.:

$$\dot{x}_c = A_c x_c + B_1 u f\left(\frac{t}{\epsilon}\right) \ , \quad x_c \in \mathbb{R}^m, \ u \in \mathbb{R}$$

where

$$A_c = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ -a_m & -a_{m-1} & \cdots & -a_1 \end{bmatrix} , \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} ,$$

and a_i are the coefficients of the characteristic polynomial of matrix A_c. Apply state feedback

$$u = Kx_c = \frac{K_1}{\epsilon}x_c = \left[\frac{k_m}{\epsilon} \cdots \frac{k_2}{\epsilon}\ 0\right] \ , \tag{2.4}$$

where $k_i \sim 1$, $i=2, \ldots, m$. In the fast time $\tau = t/\epsilon$, the closed loop system is

$$\frac{dx_c}{d\tau} = (\epsilon A_c + B_1 K_1 f(\tau)) x_c \ . \tag{2.5}$$

4

Let $\Phi(t/\epsilon)$ be a fundamental matrix for $(1/\epsilon)BK_1 f(t/\epsilon)$. Reducing (2.5) into the standard form [9] and then applying the averaging principle we have the averaged system's equation

$$\dot{\bar{x}}_c = \overline{\Phi^{-1}\left(\frac{t}{\epsilon}\right)A_c\Phi\left(\frac{t}{\epsilon}\right)}x_c \qquad (2.6)$$

where

$$\overline{\Phi^{-1}\left(\frac{t}{\epsilon}\right)A_c\Phi\left(\frac{t}{\epsilon}\right)} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ -a_m - k_2 k_m \overline{\phi^2} & -a_{m-1} - k_2 k_{m-1}\overline{\phi^2} & \cdots & -a_1 \end{bmatrix},$$

and

$$\phi\left(\frac{t}{\epsilon}\right) = \int_0^{t/\epsilon} f(\tau)d\tau .$$

Let $\lambda_1, \ldots, \lambda_m$ denote the open loop eigenvalues of (2.2) and choose the closed-loop eigenvalues in the following manner:

$$\overline{\lambda_i} = \frac{1}{m}\sum_{i=1}^{m}\lambda_i + j\,\mathrm{Im}\,\lambda_i . \qquad (2.7)$$

We determine the coefficients $\overline{a_i}$ of the closed-loop characteristic equations corresponding to $\overline{\lambda_1}, \ldots, \overline{\lambda_m}$. The state feedback gains (2.4) can be found by

$$k_i = \frac{(\overline{a_i} - a_i)}{k_2} , \quad i = 2, \ldots, m . \qquad (2.8)$$

The control gains (2.8) guarantee the asymptotic stability of the averaged system (2.6). As it follows [8], if (2.6) is asymptotically stable, there exists $\epsilon_0 > 0$ such that for all $0 < \epsilon \leq \epsilon_0$ equation (2.2) is also asymptotically stable. This proves the sufficiency. **Q.E.D.**

3. OUTPUT FEEDBACK

Theorem 3.1: There exists ϵ_0 such that for all $0 < \epsilon \leq \epsilon_0$ system (1.5) is stabilizable by an output feedback (1.7) if (A, B, C) is controllable and observable and $\mathrm{Tr}\,A < 0$. The separation principle holds, i.e. the choice of K and L can be carried out independently.

Proof: Consider system (1.5) with (A, B, C) controllable and observable, the observer,

$$\dot{\hat{x}} = A\hat{x} + Buf\left(\frac{t}{\epsilon}\right) + L(y - C\hat{x}) ,$$

and the feedback law

$$u = K\hat{x} = \frac{K_1}{\epsilon}\hat{x} .$$

In fast time $\tau = t/\epsilon$, the resulting closed-loop equations are:

$$\begin{bmatrix} \frac{dx}{d\tau} \\ \frac{d\hat{x}}{d\tau} \end{bmatrix} = \begin{bmatrix} \epsilon A & BK_1 f(\tau) \\ \epsilon LC & \epsilon(A - LC) + BK_1 f(\tau) \end{bmatrix} = \begin{bmatrix} x \\ \hat{x} \end{bmatrix} . \tag{3.1}$$

Let $\Phi(t/\epsilon)$ be a fundamental matrix for $(1/\epsilon)BK_1 f(t/\epsilon)$. Define

$$\Phi_1\left(\frac{t}{\epsilon}\right) = \Phi^{-1}\left(\frac{t}{\epsilon}\right) A\Phi\left(\frac{t}{\epsilon}\right) .$$

Reducing (3.1) into the standard form [9], and applying the averaging principle, we obtain the following averaged equations:

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\bar{\hat{x}}} \end{bmatrix} = \begin{bmatrix} A + \overline{\Phi}LC - LC & \overline{\Phi}_1 - A + LC - \overline{\Phi}LC \\ \overline{\Phi}LC & \overline{\Phi}_1 - \overline{\Phi}LC \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{\hat{x}} \end{bmatrix} . \tag{3.2}$$

To simplify (3.2), introduce the following transformation

$$\begin{bmatrix} \bar{x} \\ \bar{e} \end{bmatrix} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{\hat{x}} \end{bmatrix} ,$$

which yields

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\bar{e}} \end{bmatrix} = \begin{bmatrix} \overline{\Phi}_1 & -\overline{\Phi}_1 + A - LC + \overline{\Phi}LC \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{e} \end{bmatrix} , \tag{3.3}$$

where e is the innovations error. Thus, the separation principle holds: the eigenvalues of the averaged closed-loop system are the union of those of $\overline{\Phi}_1$ and those of $A - LC$. Using construction similar to (2.6)-(2.8), one can compute the state feedback gain required to stabilize $\overline{\Phi}_1$ and assign all the eigenvalues of $A - LC$ through the choice of L so that the stability of the closed-loop system is guaranteed. **Q.E.D.**

6

4. POLE PLACEMENT CAPABILITIES

Consider again system (1.5) with feedback (1.6) and assume that

$$K = \frac{K_1}{\epsilon} = \left[\frac{k_n}{\epsilon} \cdots \frac{k_1}{\epsilon} \right] ,$$

where $k_i \sim 1$, $i = 1, \ldots, n$. Thus, the closed loop system is

$$\dot{x} = \left(A + B \frac{K_1}{\epsilon} f \left(\frac{t}{\epsilon} \right) \right) x . \tag{4.1}$$

Using the averaging theory [9], this equation can be reduced to the averaged equation,

$$\dot{\bar{x}} = (A + \overline{B}) \bar{x} , \tag{4.2}$$

and, as it follows from [9] and [10], (4.1) is asymptotically stable if (4.2) is asymptotically stable. The stability properties of (4.2) can be checked using the following:

Theorem 4.1: Assume A and B are in the controller canonical form. Then there exists ϵ_0 such that for all $0 < \epsilon \leq \epsilon_0$,

$$\overline{B} = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ & & & \\ 0 & \cdots & 0 & 0 \\ -\overline{b_n} & \cdots & -\overline{b_2} & 0 \end{bmatrix} \tag{4.3}$$

and

$$\overline{b_i} = \sum_{s=1}^{\infty} \alpha_s^2 \frac{k_2 \, k_i}{2s^2} ,$$

where α_s, $s = 1, 2, \ldots$, are the Fourier coefficients of $f(\tau)$, i.e.

$$f(\tau) = \sum_{s=1}^{\infty} \alpha_s \sin(s\tau + \varphi s) .$$

Proof: Follows directly from Theorem 3 of [5].   **Q.E.D.**

It follows from Theorem 4.1 that the characteristic polynomials of A and $(A + \overline{B})$ have respectively, the form

$$p_0(s) = s^n + a_{10} s^{n-1} + a_{20} s^{n-2} + \cdots + a_{n0} , \tag{4.4}$$

$$p(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n , \tag{4.5}$$

7

where

$$a_1 = a_{10} , \qquad (4.6)$$

$$a_2 \geq a_{20} , \qquad (4.7)$$

and a_j, $3 \leq j \leq n$ can be arbitrarily assigned. Below we analyze to what extent the constraints (4.6), (4.7) prevent the control designer from assigning the closed loop eigenvalues in a desired region of the complex plane. More specifically, considering the closed region $D(\sigma, \phi)$ of Figure 4.1, our purpose is to identify the conditions under which we can find n complex numbers $\lambda_1, \ldots, \lambda_n$, occurring as pairs of complex poles, such that they are the roots of the polynomial (4.5) which satisfies (4.6), (4.7). When this is possible, we say that pole assignment to the region $D(\sigma, \phi)$ using vibrational feedback is possible.

As preliminary results, we have the following

Lemma 4.1: The pole assignment in the region $D(\sigma, \phi)$ using closed loop vibrational control is possible, only if

$$- a_{10} \leq n\sigma . \qquad (4.8)$$

Proof: If $\lambda_1, \ldots, \lambda_n \in D(\sigma, \phi)$ are the roots of the polynomial (4.5), then

$$- a_1 = \sum_{i=1}^{n} \lambda_i , \qquad (4.9)$$

and

$$\mathrm{Re}(\lambda_i) \leq \sigma . \qquad (4.10)$$

Equation (4.8) is obtained by equating the real parts in (4.9) and using (4.10). **Q.E.D.**

Lemma 4.2: Consider the optimization problem: Find vector $r \in I\!\!R^m$ such that

$$P(r) = \sum_{\substack{i,j=1 \\ i<j}}^{m} r_i r_j \qquad (4.11)$$

8

is maximized subject to

$$\sum_{i=1}^{m} r_i = a .$$
(4.12)

A global solution r^* to this problem is given by

$$r_1^* = r_2^* = \ldots = r_m^* = \frac{a}{m} ,$$
(4.13)

and

$$P(r^*) = \frac{(m-1)}{2m} a^2 .$$
(4.14)

Proof: Since the constraint (4.12) is always regular, we apply the Lagrange multiplier rule. The Lagrangrian is

$$L(r, \lambda) = \sum_{\substack{i,j=1 \\ i<j}}^{n} r_i r_j + \lambda \left(\sum_{i=1}^{m} r_i - a \right)$$
(4.15)

The first order necessary conditions

$$\frac{\partial L(r, \lambda)}{\partial r} = 0 ,$$
(4.16)

$$\frac{\partial L(r, \lambda)}{\partial \lambda} = 0 ,$$
(4.17)

yield a linear system of equations whose solution is (4.13) together with:

$$\lambda = \frac{1-m}{m} a .$$
(4.18)

The second order conditions ensure that (4.13) is a strict maximum. Moreover, since under the constraint (4.12) the cost function (4.11) is quadratic, this maximum is the global maximum, and the proof is complete. **Q.E.D.**

We can now state the main result:

Theorem 4.2: The real pole assignment in the region $D(\sigma, \phi)$ is possible using vibrational feedback if and only if

$$-a_{10} \leq n\sigma ,$$
(4.19)

$$a_{20} \leq \frac{n-1}{2n} a_{10}^2 .$$
(4.20)

9

Proof: The necessity of (4.19) follows from Lemma 4.1. The necessity of (4.20) follows from the fact that when $r_1, \ldots, r_n$ are the roots of (4.5), the coefficients a_1 and a_2 are

$$a_1 = -\sum_{i=1}^{n} r_i , \tag{4.21}$$

$$a_2 = \sum_{\substack{i,j=1 \\ i>j}}^{n} r_i r_j . \tag{4.22}$$

The maximum value that (4.22) can achieve subject to the constraint (4.21), (4.6) is given by Lemma 4.2 and is exactly the right hand side of (4.20). Therefore if (4.20) is violated, pole assignment to the region $D(\sigma, \phi)$ with real poles is not possible.

To prove the sufficiency of (4.19), (4.20), assume they both hold. Choose

$$r_1 = r_2 = \ldots = r_n = \frac{a_{10}}{n} . \tag{4.23}$$

It is immediately checked that these real numbers solve the problem of pole assignment to the region $D(\sigma, \phi)$, which completes the proof. **Q.E.D.**

Theorem 4.2 gives a simple solution of the problem of pole assignment to the region $D(\sigma, \phi)$ with *real* poles. When non real poles are allowed, more complicated results similar to Theorem 4.2 can be derived, and will be included in a more entensive version of this paper.

5. A CONCLUDING REMARK

As it follows from the above and [5], the conditions of closed loop vibrational stabilizability are remarkably similar to the conditions of open loop vibrational stabilizability: the only difference is that the former requires the stabilizability property of (A, B). Roughly speaking, the reason for this is that a controllable pair can always be transformed into the controllable canonical form and this transformation removes the constraint on the structure of the input matrix B.

References

[1] E.R. Wood, R.W. Powers, J.H. Cline, and C.E. Hammond, "On Developing and Flight Testing Higher Harmonic Control Systems," *Journal of the American Helicopter Society*, Vol. 30, No. 1, pp. 3-10, January 1985.

[2] G. Lehmann, "The Effect of Higher Harmonic Control (HHC) on the Four-Bladed Hingeless Model Rotor," *Vertica*, Vol. 9, No. 3, pp. 273-284, 1985.

[3] F.K. Straub and E.V. Byrns, Jr., "Application of Higher Harmonic Blade Feathering on the OH-6A Helicopter for Vibration Reduction", *NASA Contract Report* 4031, December 1986.

[4] A. Clinar, J. Deng, S.M. Meerkov, and X. Shu, "Vibrational Control of an Exothermic Reaction in a CSTR: Theory, Simulations, Experiments," *AICLE Journal*, Vol. 33, pp. 353-365, March 1987.

[5] S.M. Meerkov, "Principle of Vibrational Control: Theory and Applications," *IEEE Trans. Automat. Contr.*, Vol. AC-25, pp. 755-762, August 1980.

[6] S.M. Meerkov, "Vibrational Stabilizability of Distributed Parameter Systems," *J. Math Anal. & Appl.*, Vol. 98, pp. 408-418, 1984.

[7] R. Bellman, J. Bentsman, and S.M. Meerkov, "Vibrational Control of Non-linear Systems: Vibrational Stabilizability," *IEEE Trans. Automat. Contr.*, Vol. AC-31, pp. 710-716, August 1986.

[8] S. Lee, S.M. Meerkov, and T. Runolfsson, "Vibrational Feedback Control: Zeros Placement Capabilities," *IEEE Trans. Automat. Contr.*, Vol. AC-32, pp. 604-611, July 1987.

[9] N.N. Bogoliubov and Y.A. Mitropolsky, *Asymptotic Methods in the Theory of Nonlinear Oscillations*, New York: Gordon and Breach, 1961.

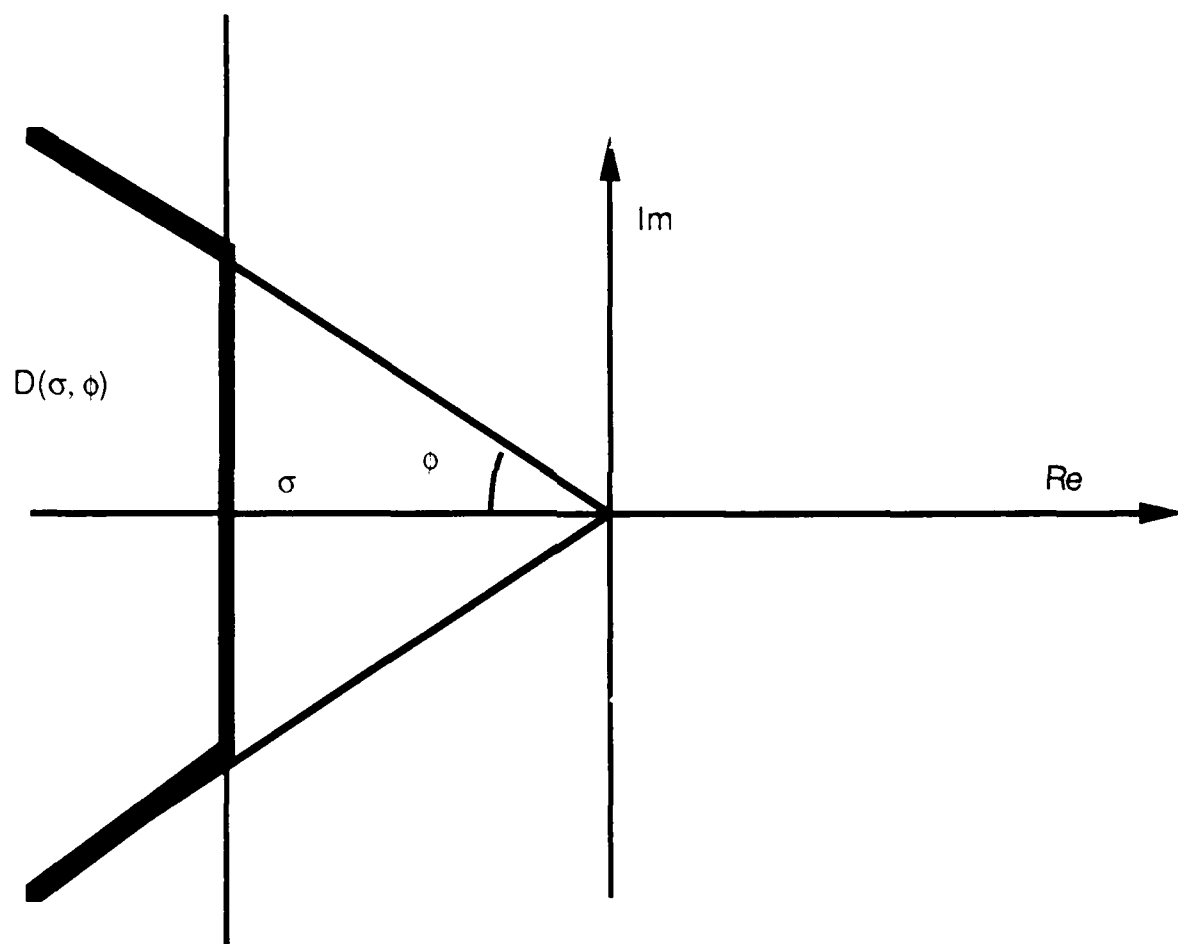[10] R. Bellman, J. Bentsman, and S.M. Meerkov, "Stability of fast periodic systems," *IEEE Trans. Automat. Contr.*, Vol. AC-31, pp. 289–291, March 1985.

Fig. 4.1 Pole Assignment Region